INDEFINITE INTEGRATION

1. If f(x) and g(x) are two functions such that $f'(x) = g(x)$ then f(x) is called antiderivative of g(x) with respect to x.
2. If f(x) is an antiderivative of g(x) then f(x) + c is also an antiderivative of g(x) for all $c \in R$.
3. If F(x) is an antiderivative of f(x) then F(x) + c, $c \in R$ is called ***indefinite integral*** of f(x) with respect to x. It is denoted by $\int f(x)\,dx$. The real number c is called ***constant of integration***.
4. The integral of a function need not exists. If a function f(x) has integral then f(x) is called an ***integrable function***.
5. The process of finding the integral of a function is known as ***Integration***.
6. The integration is the reverse process of differentiation.
7. If $n \neq -1$, then $\int x^n dx = \frac{x^{n+1}}{n+1} + c$.
8. $\int dx = x + c$
9. $\int \frac{1}{\sqrt{x}} dx = 2\sqrt{x} + c$.
10. $\int \frac{1}{x} dx = \log|x| + c$
11. $\int e^x dx = e^x + c$
12. $a > 0$, $a \neq 1$, then $\int a^x dx = \frac{a^x}{\log a} + c$
13. $\int \cos x\,dx = \sin x + c$
14. $\int \sin x\,dx = -\cos x + c$
15. $\int \sec^2 x\,dx = \tan x + c$
16. $\int \operatorname{cosec}^2 x\,dx = -\cot x + c$
17. $\int \sec x \tan x\,dx = \sec x + c$
18. $\int \operatorname{cosec} x \cot x\,dx = -\operatorname{cosec} x + c$
19. $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c = -\cos^{-1} x + c$
20. $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c = -\cot^{-1} x + c$

21. If x > 1, then $\int \frac{1}{x\sqrt{x^2-1}}\, dx = \sec^{-1} x + c =$ $-$ cosec^{-1}x + c and if x < –1 then $\int \frac{1}{x\sqrt{x^2-1}} dx = -\sec^{-1} x + c = \operatorname{cosec}^{-1} x + c$.

22. $\int \sinh x\, dx = \cosh x + c$

23. $\int \cosh x dx = \sinh x + c$

24. $\int \operatorname{sech}^2 x\ dx = \tanh x + c$

25. $\int \operatorname{cosech}^2 x\ dx = -\coth x + c$

26. $\int \operatorname{sech} x \tanh x\ dx = -\operatorname{sech} x + c$

27. $\int \operatorname{cosech} x \coth x\ dx = -\operatorname{cosech} x + c$

28. $\int \frac{1}{\sqrt{x^2-1}} dx = \sinh^{-1} x + c$

29. $\int \frac{1}{\sqrt{1-x^2}} dx = \cosh^{-1} x + c$

30. If f(x) is an integrable function and k is a real number then $\int (kf)(x)\ dx = k\int f(x)\ dx$.

31. f(x), g(x) are two integrable functions then $\int (f+g)(x)\ dx = \int f(x)\ dx + \int g(x)\ dx$.

32. If f(x), g(x) are two integrable functions then $\int (f-g)(x)\ dx = \int f(x)\ dx - \int g(x)\ dx$.

33. If $f_1(x), f_2(x), \ldots f_n(x)$ are integrable functions then $\int (f_1 + f_2 + \ldots + f_n)(x)\ dx =$ $\int f_1(x)dx + \int f_2(x)dx + \ldots + \int f_n(x)dx$.

34. If f(x), g(x) are two inegrable functions and k, *l* are two real numbers then $\int (kf + lg)(x)\ dx = k\int f(x)\ dx = l\int g(x)\ dx$

35. If $\int f(x)\ dx = g(x)$ then $\int f(ax+b)\ dx = \frac{1}{a} g(ax+b) + c$.

36. If f(x) is a differentiable function then $\int \frac{f'(x)}{f(x)}\ dx$ = log |f(x)| + c.

37. $\int \tan x$ dx = log |sec x| + c.

38. $\int \cot x$ dx = log |sin x| + c.

39. $\int \sec x$ dx=log |secx+tanx|+c = log |tan(π/4+x/2)|+c

40. $\int \operatorname{cosec} x$ dx = log |cosecx – cot x| + c = log |tan x/2| + c

41. If f(x) is differentiable function and $n \neq -1$, then $\int [f(x)]^n f'(x)dx = \frac{[f(x)]^{n+1}}{n+1} + c$.

42. If n = –1, then $\int [f(x)]^n f'(x)dx = \int \frac{f'(x)}{f(x)}dx = \log|f(x)| + c$.

43. $\int \frac{f'(x)}{\sqrt{f(x)}}dx = 2\sqrt{f(x)} + c$

44. $\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

45. $\int \frac{1}{\sqrt{a^2 + x^2}}\,dx = \sinh^{-1}\left(\frac{x}{a}\right) + c$

46. $\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \cosh^{-1}\left(\frac{x}{a}\right) + c$

47. $\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

48. $\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

49. $\int \frac{1}{x^2 - a^2}\,dx = \frac{1}{2a}\log\left|\frac{x-a}{x+a}\right| + c$

50. $\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

51. $\int \sqrt{a^2 + x^2}\,dx = \frac{x}{2}\sqrt{a^2 + x^2} + \frac{a^2}{2}\sinh^{-1}\left(\frac{x}{a}\right) + c$

52. $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\cosh^{-1}\left(\frac{x}{a}\right) + c$

53. If the given integral is of the form $\int \frac{px+q}{ax^2+bx+c}dx$ then take px + q = $A\frac{d}{dx}(ax^2+bx+c) + B$.

54. If the given integral is of the form $\int \frac{px+q}{\sqrt{ax^2+bx+c}}dx$ then take px + q = $A\frac{d}{dx}(ax^2+bx+c) + B$.

55. If the given integral is of the form $\int (px+q)\sqrt{ax^2+bx+c}\,dx$ then take px + q = $A\frac{d}{dx}(ax^2+bx+c) + B$.

56. If the given integral is of the form $\int \frac{1}{(px+q)\sqrt{ax^2+bx+c}}\,dx$, then put px + q = $\frac{1}{t}$.

57. If the given integral is of the form $\int \frac{1}{(ax^2+b)\sqrt{cx^2+d}}\,dx$ then put x = $\frac{1}{t}$.

58. If the given integral is of the form $\int \frac{px+q}{\sqrt{ax+b}}dx$ or $\int \frac{\sqrt{ax+b}}{px+q}dx$ or $\int (px+q)\sqrt{ax+b}\ dx$ or $\int \frac{1}{(px+q)\sqrt{ax+b}}$ dx then put ax + b = t^2 and hence dx = $\frac{1}{a}$2t dt.

59. If the integral is of the form $\int \frac{dx}{a^2\cos^2 x + b^2\sin^2 x}$ or $\int \frac{dx}{a\cos^2 x + b\sin x\cos x + c\sin^2 x}$ then multiply both numerator and denominator with $\sec^2$x and take tan x = t.

60. If the integral is of the form $\int \frac{dx}{a+b\cos x}$ or $\int \frac{dx}{a+b\sin x}$ or $\int \frac{dx}{a\cos x + b\sin x + c}$, take $\tan\frac{x}{2} = t \Rightarrow$ $\frac{1}{2}\sec^2\frac{x}{2}dx = dt \Rightarrow (1+\tan^2 x/2)dx = 2dt \Rightarrow$

$(1 + t^2)$ dx = 2 dt $\Rightarrow$ dx = $\frac{2dt}{1+t^2}$.

Sin x = $\frac{2\tan x/2}{1+\tan^2 x/2} = \frac{2t}{1+t^2}$,

cos x = $\frac{1-\tan^2 x/2}{1+\tan^2 x/2} = \frac{1-t^2}{1+t^2}$.

61. If the integral is of the form $\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}dx$, take acos x + b sin x = A $\frac{d}{dx}$ (c cos x + d sin x) + B(c cos x + d sin x).

62. **Integration by Parts :** If f(x) and g(x) are two integrable functions then $\int f(x).g(x)\ dx = f(x)\int g(x)\ dx - \int f'(x)\cdot[\int g(x)\ dx]\ dx$.

63. If u and v are two functions of x then $\int u\ dv = uv - \int v\ du$.

64. If u and v are two functions of x; u′, u″, u‴ … denote the successive derivatives of u and v_1, v_2, v_3, … denote the successive integrals of v then the extension of Integration by parts is $\int u\ v\ dx$ = $uv_1 - u'v_2 + u''\ v_3 - u'''v_4 + \ldots\ldots.$

65. In integration by parts, the first function will be taken as in the following order. Inverse functions, logarithmic functions, Algebraic functions, Trigonometric functions and exponential functions. (To remember this a phrase ILATE).

66. $\int e^{ax}\cos bx\ dx = \frac{e^{ax}}{a^2+b^2}$ (a cos bx + b sin bx) + c.

67. $\int e^{ax}\sin bx\ dx = \frac{e^{ax}}{a^2+b^2}$ (a sin bx – b cos bx)

68. $\int e^x[f(x)+f'(x)]dx = e^x f(x) + c$.

69. $\int e^{-x}[f(x)-f'(x)]\ dx = -e^{-x}f(x) + c$

70. If $I_n = \int \sin^n x\,dx$ then $I_n = \frac{-\sin^{n-1}x\cos x}{n} + \frac{n+1}{n}I_{n-2}$, where n is the +ve integer.

71. If $I_n = \int \cos^n x\,dx$ then $I_n = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} I_{n-2}$.

72. If $I_n = \int \tan^n x\,dx$ then $I_n = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$.

73. If $I_n = \int \cot^n x\,dx$ then $I_n = \frac{-\cot^{n-1} x}{n-1} - I_{n-2}$.

74. If $I_n = \int \sec^n x\,dx$ then $I_n = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} I_{n-2}$.

75. If $I_n = \int \operatorname{cosec}^n x\,dx$ then $I_n = \frac{-\operatorname{cosec}^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} I_{n-2}$.

76. If $I_n = \int (\log x)^n\,dx$ then $I_n = x(\log x)^n - nI_{n-1}$.

77. If $I_{m,n} = \int \sin^m x \cos^n x\,dx$, then

$$I_{m,n} = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} I_{m,n-2} = -\frac{\sin^{m-1} x \cos^{n+1} x}{m+n} + \frac{m-1}{m+n} I_{m-2,n}$$

78. $\int f(ax+b)\,dx = \frac{1}{a}\int f(t)\,dt + c$, where t = ax + b.

79. $\int \frac{f'(x)}{f(x)}\,dx = \log|f(x)| + c$

80. $\int [f(x)]^n f'(x)\,dx = \frac{[f(x)]^{n+1}}{n+1} + c$.

81. $\int f(x^n) x^{n-1}\,dx = \frac{1}{n}\int f(t)\,dt$, where $t = x^n$.

82. $\int \frac{a\cos x + b\sin x}{c\cos x + d\sin x}\,dx = \frac{ac+bd}{c^2+d^2} x + \frac{ad-bc}{c^2+d^2} \log|c\cos x + d\sin x| + c$.

83. If a > b then $\int \frac{dx}{a + b\cos x} = \frac{2}{\sqrt{a^2-b^2}} \tan^{-1}\left[\sqrt{\frac{a-b}{a+b}} \tan\frac{x}{2}\right] + c$.

84. $\int e^x [f(x) + f'(x)]\,dx = e^x f(x) + c$

85. $\int e^{ax}\left[f(x) + \frac{f'(x)}{a}\right] dx = \frac{e^{ax} f(x)}{a} + c$

86. $\int [xf'(x) + f(x)]\,dx = xf(x) + c$.

87. $\int e^{ax} \sin(bx+c)\,dx = \frac{e^{ax}}{a^2+b^2}$ [a sin (bx + c) –
b cos (bx + c)] + k.

88. $\int e^{ax} \cos(bx+c)\,dx = \frac{e^{ax}}{a^2+b^2}$ [a cos (bx + c) +
b sin (bx + c)] + k.

89. $\int xe^{ax}\sin(bx+c)dx = \frac{xe^{ax}}{a^2+b^2}$ [a sin (bx + c) –

b cos (bx + c)] – $\frac{e^{ax}}{(a^2+b^2)^2}[(a^2-b^2)\sin bx \ -2ab\cos bx]+k$.

90. $\int xe^{ax}\cos(bx+c)dx = \frac{xe^{ax}}{a^2+b^2}$ [a cos (bx + c) +

b sin (bx + c)] – $\frac{e^{ax}}{(a^2+b^2)^2}[(a^2-b^2)\cos(bx+c)+\ 2ab\sin(bx+c)]+k$.